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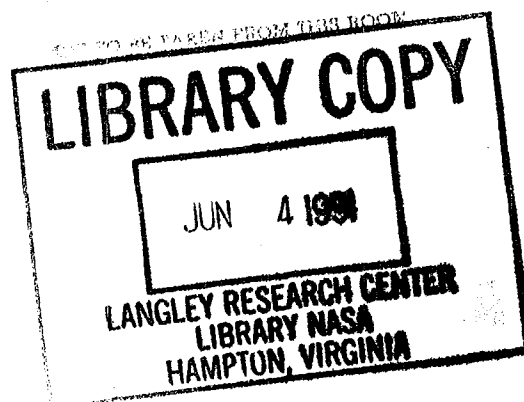
NASA TECHNICAL MEMORANDUM

NASA-TM-75815

IV. BOUNDARY LAYERS AND RESISTANCE
ON LIQUID MOTION WITH ONLY SLIGHT FRICTION
(no author given)

Translation of "IV. Grenzsichten und Widerstand - Über Flüssigkeitsbewegung bei sehr kleiner Reibung," International Mathematical Congress, 3rd, Heidelberg, 1904 Leipzig: Teubner, 1905, pp 484-491

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16. Abstract The aim of this study is to examine the laws of fluid motion systematically, where friction is assumed very slight. Calculations are carried out with the appropriate differential equation and practical investigations are illustrated.			
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IV. BOUNDARY LAYERS AND RESISTANCE

(no author given)

On Fluid Motion with Slight Friction

/575*

Standard hydrodynamics deals mainly with the motion of fluids without friction. The fluid friction is calculated by means of the differential equation of motion, the application of which has been confirmed by physical observations. Only such solutions for this differential equation are available, in which the fluid inertia is disregarded or plays no role, in addition to the one-dimensional problems, as were presented by Lord Rayleigh¹. The solution for the two and three-dimensional problem with friction and inertia as factors has yet to be found. The reason for this probably lies in the problematic characteristics of the differential equation. This is expressed in Gibbs vector symbols² as:

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) + \nabla (V + p) = k \nabla^2 \mathbf{v} \quad (1)$$

(\mathbf{v} is velocity, ρ density, V function of forces, p pressure, k coefficient of friction); the equation of continuity is also applied; for incompressible fluids, the only type to be treated here, this becomes simply

$$\text{div } \mathbf{v} = 0.$$

*Numbers in the margin indicate pagination in the foreign text.

¹ Proc. Lond. Math. Soc. 11, P. 57 = Papers I, P. 474f.

² $\mathbf{a} \cdot \mathbf{b}$ linear product, $\mathbf{a} \times \mathbf{b}$ vector product, ∇ Hamilton Differential operator

$$\left(\nabla = \mathbf{i} \frac{\partial}{\partial x} + \mathbf{j} \frac{\partial}{\partial y} + \mathbf{k} \frac{\partial}{\partial z} \right).$$

It can be easily seen from the differential equation that the factor η compared to the other factors becomes indefinitely small with sufficiently slow and even slowly altered motions, so that the effect of inertia may be disregarded in a close approximation. On the other hand, when the motion is sufficiently rapid, the squared factor of v ($v \cdot \nabla v$ (alteration in velocity as a result of position change) is large enough to reduce the influence of friction $k \nabla^2 v$ to a minor role. This almost always holds true for cases of fluid motion in technology. Therefore it seems logical in this case simply to employ the equation for fluids /576 without friction; however, information gathered shows that the known solutions for this equation usually are in poor agreement with experience - the Dirichlet sphere can be mentioned in this connection, which should move with no resistance according to theory.

The aim of this study is to examine the laws of fluid motion systematically, where friction is assumed very slight. The friction should be so slight that it may be disregarded everywhere, where there is no large difference in velocity and where no cumulative effect of friction is registered. This plan has proven very productive, since on the one hand mathematical formulations result, making possible solutions for the problems, and on the other hand a satisfactory agreement with the observations is probable. First it should be mentioned that in the transition from motion with friction to the limit of no friction in the case of stationary motion around a sphere, the result is very different from the Dirichlet motion. The Dirichlet motion is only a beginning state, which is immediately disturbed by the effects of even a small amount of friction.

Now we can begin with the individual questions. The force stemming from friction on the unit cube is

$$K = k \nabla^2 v;$$

when $w = \frac{1}{2} \text{ red } v$ designates the vortex, then $K = -2k \text{ red } w$ according to a known vector-analytical conversion, taking into consideration that

$\text{div } v = 0$. From this it of course results that K is also 0 when w is 0, i.e. that the irrotational motion represents a possible motion even in the case of randomly great friction; however, when this does not result in certain cases, the cause is the shift of fluid turbulence from the edge into the irrotational fluid.

The effect of friction, even when it is very small, may accumulate in the case of a random periodical or cyclical motion of longer duration.

It must therefore be required for the inertial state that the work of K , i.e. the line integral $\int K \circ ds$, is equal to 0 for a full cycle along each flow line with cyclical motion; the equation for the periods in the case of periodic flows according to location;

$$\int_P K \circ ds = (V_2 + p_2) - (V_1 + p_1).$$

In the case of two-dimensional motion with a flow function ψ ³, a general expression on the distribution of turbulence may be derived /577 from this with the aid of the Helmholtz laws of turbulence. The equation obtained for smooth motion is⁴

$$-\frac{dw}{d\psi} = \frac{(V_2 + p_2) - (V_1 + p_1)}{2k \int v \circ ds};$$

This is equal to zero in the case of closed flow lines; this then provides the simple result that within an area of closed flow lines the vortex assumes a constant value.

³ Compare Enzyklop. math. Wiss. IV, 14, 7.

⁴ According to Helmholtz the turbulence of a partical is constantly proportional to the length in the direction of vortex axis; in the stationary smooth motion w is constant for each flow line ($\psi = \text{constant}$), therefore $w = f(\psi)$; it follows that

$$\int K \circ ds = 2k \int \text{rot } w \circ ds = 2k f'(\psi) \int \text{rot } \psi \circ ds = 2k f'(\psi) \int v \circ ds.$$

In the case of axially symmetrical motion with flow on the meridian plane the vortex is proportional to the radius for closed flow lines: $w = cr$, resulting in a force $K = 4kc$ in the direction of the axis.

The most important problem segment is the behavior of the fluid at the walls of the stationary body. Sufficient consideration is made of the physical processes in the boundary layer between fluid and fixed body by assuming that the fluid adheres at the walls, therefore that the velocity is equal to zero at that location, i.e. equal to the velocity of the body. If the friction is very slight and the path of the fluid along the wall not very long, the velocity attains a normal value already very close to the wall. The abrupt differences in velocity in the narrow transition layer then result in noticeable effects in spite of the low coefficient of friction.

The best method for dealing with this problem is to schedule factors to be neglected in the general differential equations. If k is small from the 2nd order, the thickness of the transition layer is small from the first order, as is the normal component of velocity. The transverse differences in pressure can be disregarded, as well as a possible curvature in the flow lines. The pressure distribution is impressed on the transition layer by the free fluid.

For the problem with constant factors, which has been solely discussed here, in the stationary conditions (X direction tangential, Y direction normal, U and V the corresponding components of velocity) the resulting differential equation is

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) + \frac{dp}{dx} = k \frac{\partial^2 u}{\partial y^2};$$

with the additional equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

If dp/dx is given as usual, further the course of u for the beginning cross-section, such tasks may be calculated by means of the quadratures for extracting the corresponding $\partial u/\partial x$ from each u ; with this procedure and the assistance of one of the known methods of approximation⁵ progress can be made in the X direction step by step. One difficulty, however, consists in the various singularities occurring at the fixed edge. The simplest case of the states of motion discussed here is water flowing along a smooth thin plate. The variables can be reduced here by employing $u = f\left(\frac{y}{\sqrt{x}}\right)$. The formula for resistance is obtained by the numerical resolution of the resulting differential equation

$$R = 1,1 \dots b \sqrt{k \rho l u_0^3}$$

(b width, l length of the plate, u_0 velocity of the undisturbed water compared to the plate). Figure 1 shows the course of u .

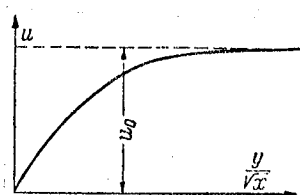


Figure 1

The most important result of these studies for practical application, however, is that in certain cases the flow completely separates from the wall at a position determined by external conditions (compare Figure 2). A fluid layer, set into rotation by friction at the wall, is pushed into the free fluid and plays the same role as the Helmholtz separation layers there, effecting a complete change in motion. With a change in coefficient of friction k only the thickness of the turbulence layer is altered (proportional to $\sqrt{\frac{k l}{\rho u}}$), everything else remains unchanged; a transition can therefore be made to the limit $k = 0$, still retaining the same flow shape.

⁵ Compare z. B. Z. Math. Phys. Vol. 46, p. 435 (Kutta).

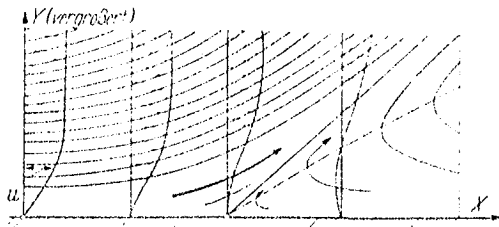


Figure 2

Key:

a. enlarged

A closer examination provides the result that the necessary conditions for the separation of stream is the presence of a pressure increase along the wall in the direction of the flow. The amount of pressure increase in certain cases can only be concluded from the evaluation, still to be conducted. A plausible reason for the separation of the flow is the partial conversion of free fluid

kinetic energy into potential energy in the case of a pressure increase. The transition layers, however, have lost a large portion of their kinetic energy; they no longer have enough to penetrate into the area of higher pressure and therefore pass by to the side.

According to the preceding, a certain flow process is divided into two portions interacting with one another. On the one hand there is the free fluid, which can be treated as free of friction according to the Helmholtz laws of turbulence, and on the other hand there are the transition layers at the fixed boundaries, the motion of which is regulated by the free fluid, which in turn gives the characteristic structure to the free motion by means of layers of turbulence.

The attempt has been made to follow the process more closely by drawing the flow lines in a few cases; however, the results make no claim to quantitative accuracy. Insofar as the motion is free of turbulence, the circumstance is employed advantageously in drawing that the flow lines form a quadratic network of curves with the lines of constant velocity potential.

Figures 3 and 4 show the beginning of the motion around a wall projecting into the flow in two stages. The beginning motion, free of turbulence, is rapidly altered by a separation layer (dotted lines), beginning at the edge of the obstacle and winding in a spiral. The turbulence moves further and further away and leaves still water behind

the finally stationary separation layer.

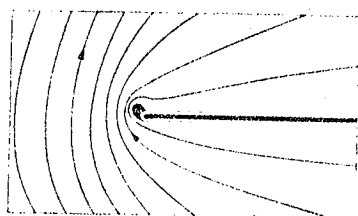


Figure 3

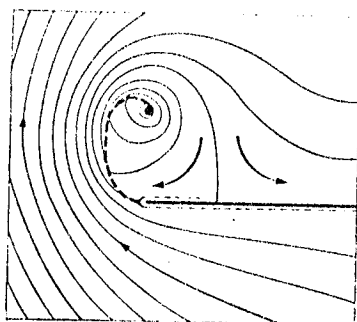


Figure 4

The analogous process in the case of a circular cylinder can be seen in Figures 5 and 6. The fluid layers set into rotation by the friction are again designated by dotted lines. The separation surfaces also extend here in the inertial state to infinity. These separation surfaces are all known to be unstable; if a small sine wave disturbance is present, motion results as presented in Figures 7 and 8. It can be seen how clearly separated turbulence is formed by the interaction of the fluid flows. The layer of turbulence is rolled up inside this vortex, as represented in Figure 9. The lines in this figure are not flow lines, but rather lines obtained, for example, by the addition of colored liquid.

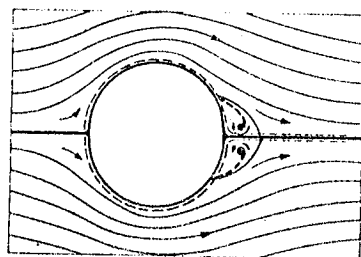


Figure 5

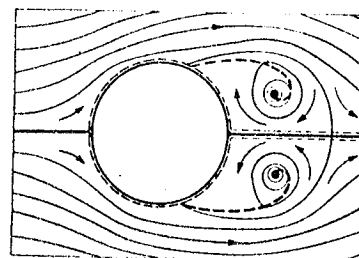


Figure 6

Now a short discussion will be presented on the investigations conducted for comparison with theory. The experimental equipment (shown in Figure 10 in a vertical and a horizontal projection) consists of a 1.5 m long tank with an intermediate base. The water is circulated by a blade wheel and enters the upper section of the water without turbulence after passing through an arrangement of guiding device a and four screens b; the object to be investigated is introduced at c. In

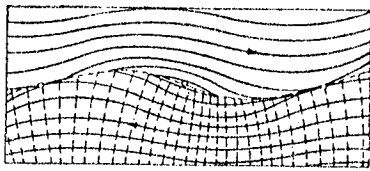


Figure 7

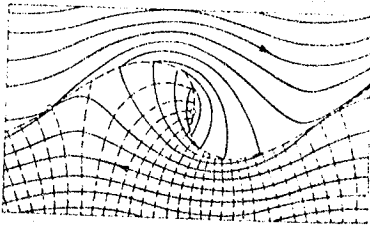


Figure 8

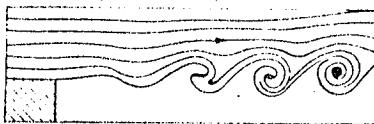


Figure 9

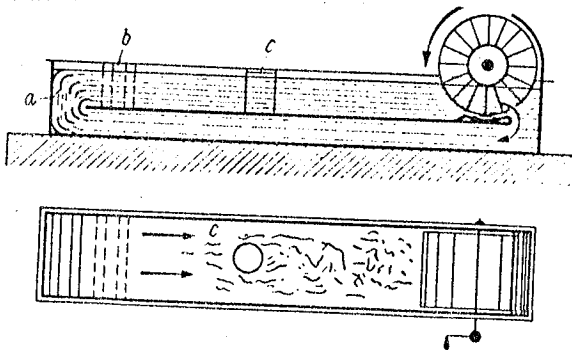


Figure 10

the water is a suspension of fine shiny iron mica foil; by means of this method a characteristic shine emphasizes all somewhat deformed points in the water, especially all turbulence, due to the orientation of the foil at these positions.

The photographs on pages 582 and 583 were obtained in this manner. In all exposures the flow goes from the left to the right. Exposure 581 No. 1 to 4 deals with the motion at a wall projecting into the flow. The separation surface can be seen, extending from the edge. This is still very small in 1, already covered with large disturbances in 2, the turbulence extends over the entire picture in 3 and the "inertial state" is shown in 4. A disturbance is also noticeable above the wall. Since there is a higher pressure in the corner as a result of the congestion of the water flow, the flow also separates from the wall here in time (compare p. 580). The various bands visible in the "turbulence free" portion of

the flow (especially in No. 1 and 2) stem from the circumstance that the liquid was not completely calm at the beginning of motion. No. 5 and 6 show the flow around a circular curved obstacle, or in other words, through a continuously decreasing and then again increasing channel. No. 5 shows a stage shortly after the beginning of the motion. One separation surface is wound to a spiral and the other is extended and separated into very regular turbulence. On the convex side close to the right end, the beginning of a separating flow can be seen. No. 6 shows



1



2



3



4



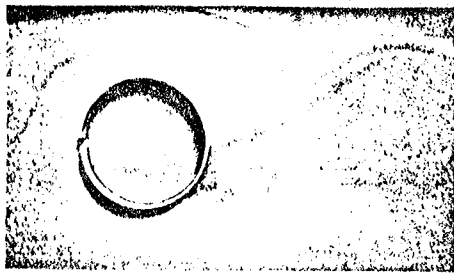
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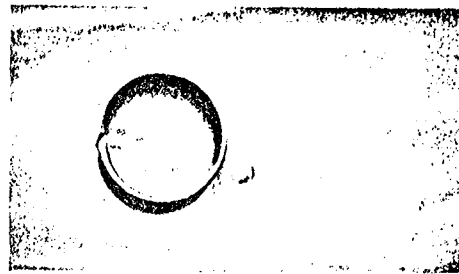
6

the inertial state, at which the flow separates at about the most narrow cross-section.

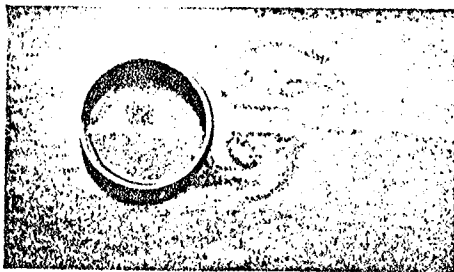
No. 7 to 10 show the flow around a circular cylinder (a hollow post). No. 7 shows the beginning of separation, No. 8 and 9 show further stages. A line is visible between the two disturbances, consisting of water which belonged to the transition layer before separation. No. 10 shows the inertial state. The tail of turbulent water behind the cylinder waves back and forth creating the unsymmetrical momentary shape. The cylinder has a gap running along a generator. If this is positioned /584 as in No. 11 and 12 and water is pumped from the cylinder with the aid



7



8



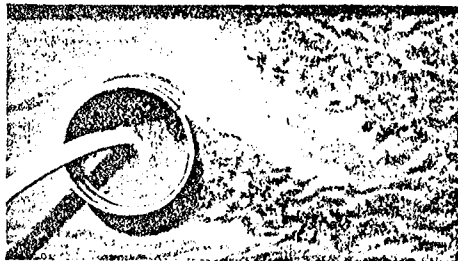
9



10



11



12

of a hose, the transition layer on one side can be intercepted. When this is missing, its effect, i.e. separation, must also be absent. In No. 11, corresponding in time to No. 9, only one disturbance and the line can be seen. In No. 12 (inertial state) the flow follows the cylinder wall closely up to the slit, although only a very small portion of the water enters into the cylinder, as can be seen. Instead a separation surface has now formed on the smooth outer wall of the tank (a first suggestion of this phenomenon may already be seen in No. 11).

Since the velocity is reduced by the expanding flow opening, causing a pressure increase⁶, the conditions for a separation of flow from the wall are given, providing an explanation for this remarkable phenomenon in the theory presented.

6

constant on every flow line.